HIGH SCHOOL AND UNIVERSITY MATRICULATION EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1942

ALGEBRA 2

Time—3 hours.

Note—All answers are to be written in this booklet.

Ample space has been provided for rough work. Page 28 may be used for rough work for the exercises of page 1.

It is not expected that you will have sufficient time to do all the questions, but answer as many as you can.

In questions 1 to 25, answers only are required. Place these in the brackets. In questions 26 to 56, complete solutions should be shown in the spaces provided.

Mathematical tables will be supplied by the presiding examiner.

Values

2 **1.** If $5y = 4x$, find the value of the ratio $y : x$. (..............................)

2 **2.** $a : b = c : d$

Write a fraction equal to $\frac{a}{b}$. (..............................)

Write a fraction equal to $\frac{a}{c}$. (..............................)

2 **3.** L varies inversely as W. How is L affected when W is halved? (..............................)

2 **4.** What value of x will make the function $\frac{x + 3}{5 - x}$ equal to zero? (..............................)

2 **5.** For what value of x is the function in question 4 not defined? (..............................)

2 **6.** Express the perimeter (p inches) of a semicircle as a function of its radius (r inches). (..............................)

2 **7.** If $f(x) = 5x - 2x^3$, find the value of $f(-2)$. (..............................)

2 **8.** As h approaches the limit 0, what limit does $\frac{5h + h^2}{h}$ approach? (..............................)

2 **9.** What is the gradient of the graph of the function $3x + 5$? (..............................)

2 **10.** Given $y = 5x^2 + 7$. Find $\frac{d^2y}{dx^2}$. (..............................)

2 **11.** At what value of x does x^2 change from a decreasing to an increasing function? (..............................)

FOR ROUGH WORK.

Values

2 **12.** Find a value of x at which the slope of the graph of $x^2 - 4x$ is equal to zero. (..)

2 **13.** $\frac{dy}{dx} \triangle x$ is often used as an approximation for $\triangle y$. For which of the following values of $\triangle x$ would you expect the approximation to be closest: 0.015, 0.021, 0.02 ? (..)

2 **14.** Solve $\frac{ds}{dt} = t^2$. (..)

2 **15.** Solve $\frac{ds}{dt} = \frac{1}{t^2}$. (..)

2 **16.** Evaluate $\int_1^3 2x\,dx$. (..)

2 **17.** Write the fifth term in the following sequence:
$\frac{1}{2}, \frac{4}{5}, \frac{9}{10}, \frac{16}{17},$ .... (..)

2 **18.** The nth term of an *A. P.* is $\frac{6n + 19}{4}$.
Write the 10th term. (..)

2 **19.** The first two terms of a *G. P.* are 15 and 10. Write the third term. (..)

2 **20.** Given $f(x) = (1.03)^x$. Use tables to find the value of $f(21)$ correct to the second decimal place. (..)

2 **21.** How many different committees consisting of one man and one woman can be chosen from a group of 5 men and 6 women? (..)

2 **22.** Evaluate $\frac{10!}{6!\ 4!}$ (..)

2 **23.** If $(x + 5)(2x + 3)(3x + 2)(5x + 1)$ is expanded and arranged in descending powers of x, what are the first and the last terms?
(first) (..)
(last) (..)

2 **24.** Using a table of logarithms, solve for x: $10^x = 37.5$. (..)

2 **25.** What is the gradient of the line joining the points (2,1) and (5,6)? (..)

[OVER]

FOR ROUGH WORK.

Values

In questions 26 to 56, complete solutions should be shown in the spaces provided.

5 **26.** The ratio of the volumes of two circular cylinders is 15 : 14 and the ratio of their heights is 5 : 21. Find the ratio of their diameters.

4 **27.** The time of oscillation of a pendulum varies as the square root of its length. A pendulum 24 inches long makes one complete oscillation in $1\frac{1}{2}$ seconds. Calculate to the nearest hundredth of an inch the length of a pendulum which makes one complete oscillation in one second.

FOR ROUGH WORK.

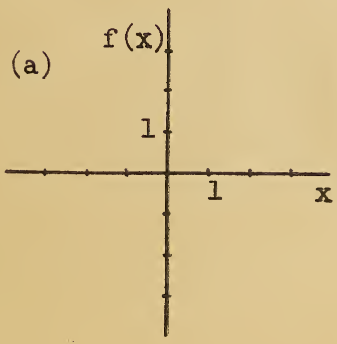
Values

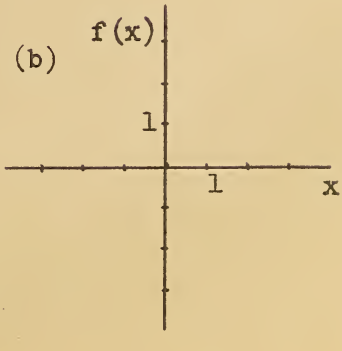
4 **28.** When $s = 2$, $V = \dfrac{1}{\sqrt{t}}$ and when $t = 4$, $V = \dfrac{s^2}{8}$

Derive a formula expressing V in terms of s and t.

5 **29.** Sketch the graph of (*a*) $\dfrac{x + 1}{x - 1}$ and (*b*) $\dfrac{x + 1}{1 - x}$.

Use the sets of axes provided below.





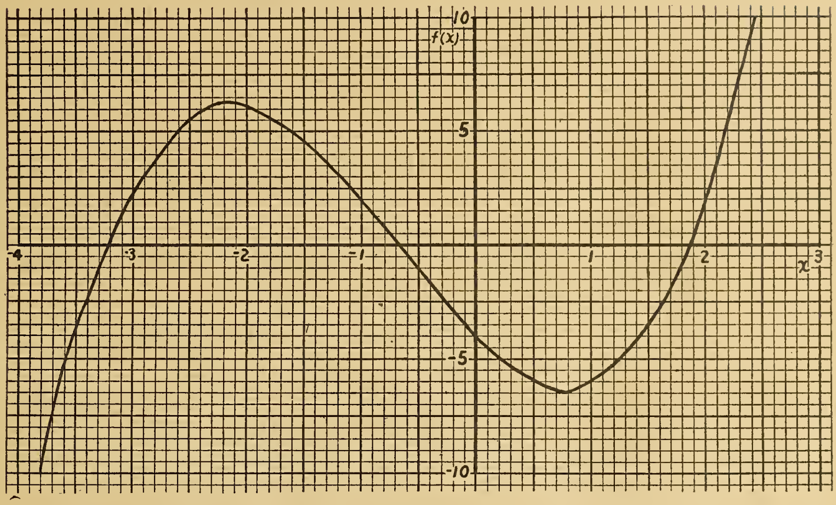
6 **30.** A right triangle has a perimeter of x inches, and one of the sides about the right angle is 4 inches long. Express the area of the triangle (A sq. in.) as a function of x.

FOR ROUGH WORK.

Values

2 **31.** The diagram below shows the graph of $x^3 + 2x^2 - 5x - 4$.

Use this graph to solve the equation $x^3 + 2x^2 - 5x - 4 = 0$. Give answers to the nearest tenth.



3 **32.** From the graph given in question 31, solve the equation $x^3 + 2x^2 - 5x + 1 = 0$. Give answers to the nearest tenth.

FOR ROUGH WORK.

Values

3 **33.** On the set of axes used for the graph in question 31, draw the graph of $2x + 1$. Show values used for plotting in the table provided below.

x	
$2x + 1$	

3 **34.** Using the original graph given in question 31 and the graph which you were directed to draw in question 33, solve the equation $x^3 + 2x^2 - 7x - 5 = 0$. Give answers to the nearest tenth.

2 **35.** Consider functions of the form $ax^2 + bx + c$. How are the graphs of such functions restricted in shape or position when $a = 0$, $b \neq 0$, $c \neq 0$?

FOR ROUGH WORK.

Values

3 **36.** As in question 35, how are the graphs restricted when $a \neq 0$, $b = 0$, $c \neq 0$?

2 **37.** As in question 35, how are the graphs restricted when $a \neq 0$, $b \neq 0$, $c = 0$?

FOR ROUGH WORK.

Values

3 **38.** For what range of values of K are the roots of the equation $2x^2 - 3x + K = 0$ real and unequal?

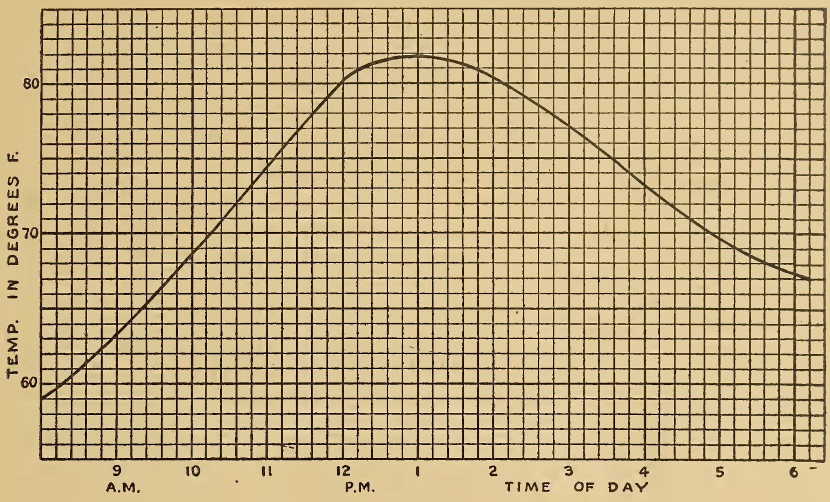
3 **39.** The function $2x^2 + x - 1$ may be expressed in the form $2(x + A)(x + B)$. Find A and B.

3 **40.** As n increases without limit, the functions $\frac{n^2}{n^2 + 1}$ and $\frac{n^2}{n^2 - 1}$ both approach the limit $+1$. How do they differ in their manner of approaching this limit?

FOR ROUGH WORK.

Values

2 **41.** The accompanying diagram shows the variations in temperature at a certain place during part of a summer day. Find the average rate of change of the temperature between 11 a.m. and 2 p.m.



3 **42.** From the diagram given in question 41, estimate the rate of change of temperature at 1:30 p.m.

4 **43.** If $y^2 = x^3$, find the value of $\frac{dy}{dx}$ when $x = 4$.

FOR ROUGH WORK.

Values

6 **44.** Find all the maximum and minimum values of $4x^3 + 15x^2 - 18x + 7$.

3 **45.** The volume of a sphere is $\frac{4}{3}\pi r^3$. If the radius of the sphere is increased from 5 cm. to 5.03 cm. use the theorem mentioned in question 13 to obtain an approximate value for the increase in the volume of the sphere. Give the answer correct to two decimal places.

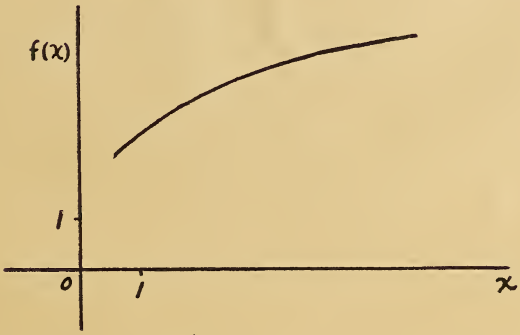
FOR ROUGH WORK.

Values

4 **46.** Given $\frac{d^2s}{dt^2} = 3t + \frac{1}{2}$. Also, when $t = 2$, $s = 6$ and $\frac{ds}{dt} = 10$.

Express s as a function of t.

2 **47.** Using the sketch below, mark the boundary of the area $\int_a^b f(x)\,dx$ with four heavy lines.



6 **48.** A sphere having a diameter of 10 inches is divided into two parts by a plane which is 3 inches from the centre of the sphere. Use integration to find the volume (correct to one decimal) of the smaller part.

FOR ROUGH WORK.

Values

2 **49.** The first term of an *A. P.* is 3 and the seventeenth term is 27. Find the sum of these seventeen terms.

5 **50.** If the sum of the first n terms of the *A. P.* in question 49 is 486, find n.

4 **51.** Find (to the nearest cent) the interest earned by investing $250.00 for 7 years at 3% per annum compounded semi-annually.

FOR ROUGH WORK.

Values

6 **52.** A \$50.00 bond bearing interest at 3% is redeemable at par in 10 years. Find to the nearest cent the cash value of this bond if the current discount rate is $2\frac{1}{2}$%.

5 **53.** There are 17 different books from which the winner of a contest may select the 3 to which he is entitled. Of these 17, 10 are fiction and 7 are non-fiction. If at least one of the books chosen must be non-fiction, in how many ways can the 3 books be selected?

FOR ROUGH WORK.

4 **54.** Write down and simplify the first four terms in the expansion of $\left(x-\frac{y}{2}\right)^{16}$.

3 **55.** Write the middle term in the expansion of the binomial named in question 54.

When $x = 2$ and $y = 1$, what is the value of this term?

3 **56.** Assuming that, under certain conditions, $(1 + x)^n \simeq 1 + nx$, find an approximate value for $\frac{1}{\sqrt{104}}$.

FOR ROUGH WORK.